Benha University **Antennas &Wave Propagation** **Electrical Eng. Dept.**
Faculty of Engineering **4^{th} year communication**
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Sheet (3)-solution

1. The maximum radiation intensity of a 90% efficiency antenna is 200 mW/ unit solid angle. Find the directivity and gain (dimensionless and in dB) when the
   (a) Input power is 125.66 mW
   (b) Radiated power is 125.66 mW

(a) $D_o = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi(200\times10^{-3})}{0.9\,(125.66\times10^{-3})} = 22.22 = 13.47\,dB$

$G_o = e_t \cdot D_o = 0.9\,(22.22) = 20 = 13.01\,dB$

(b) $D_o = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi(200\times10^{-3})}{(125.66\times10^{-3})} = 20 = 13.01\,dB$

$G_o = e_t \cdot D_o = 0.9\cdot(20) = 18 = 12.55\,dB$

2. A lossless resonant half-wavelength dipole antenna, with input impedance of 73 ohms, is connected to a transmission line whose characteristic impedance is 50 ohms. Assuming that the pattern of the antenna is given approximately by $U=B_o\sin^3\theta$. Find the maximum gain and maximum absolute gain of this antenna.

$$U|_{\max} = U_{\max} = B_0$$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta\, d\theta\, d\phi = 2\pi B_0 \int_0^{\pi}\sin^4\theta\, d\theta = B_0\left(\frac{3\pi^2}{4}\right)$$

$$D_0 = 4\pi\frac{U_{max}}{P_{rad}} = \frac{16}{3\pi} = 1.697$$

Since the antenna was stated to be lossless, then the radiation efficiency $e_{cd} = 1$.

$$G_0 = e_{cd} D_0 = 1(1.697) = 1.697$$

$$e_r = (1 - |\Gamma|^2) = \left(1 - \left|\frac{73-50}{73+50}\right|^2\right) = 0.965$$

$$G_{0abs} = e_0 D_0 = 0.965(1.697) = 1.6376$$

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3. A uniform plane wave, of is traveling in the positive z-direction. Find the polarization (linear, circular, or elliptical), sense of rotation (CW or CCW), when
(a) $E_x = E_y$, $\Delta\varphi = \varphi_y - \varphi_x = 0$ (b) $E_x \neq E_y$, $\Delta\varphi = \varphi_y - \varphi_x = 0$
(c) $E_x = E_y$, $\Delta\varphi = \varphi_y - \varphi_x = \pi/2$ (d) $E_x = E_y$, $\Delta\varphi = \varphi_y - \varphi_x = -\pi/2$
(e) $E_x = E_y$, $\Delta\varphi = \varphi_y - \varphi_x = \pi/4$ (f) $E_x = E_y$, $\Delta\varphi = \varphi_y - \varphi_x = -\pi/4$
(g) $E_x = 0.5E_y$, $\Delta\varphi = \varphi_y - \varphi_x = \pi/2$ (h) $E_x = 0.5E_y$, $\Delta\varphi = \varphi_y - \varphi_x = -\pi/2$

(a) Linear because $\Delta\phi = 0$.
(b) Linear because $\Delta\phi = 0$.
(c) Circular because 1. $E_x = E_y$
2. $\Delta\phi = \pi/2$ CCW
(d) Circular because 1. $E_x = E_y$
2. $\Delta\phi = -\pi/2$ CW
(e) Elliptical because $\Delta\phi$ is not multiples of $\pi/2$. CCW
(f) Elliptical because $\Delta\phi$ is not multiples of $\pi/2$ CW
(g). Elliptical because 1. $E_x \neq E_y$
2. $\Delta\phi$ is not zero or multiples of π.
CCW
(h) Elliptical because 1. $E_x \neq E_y$
2. $\Delta\phi$ is not zero or multiples of π.
CW

4. A wave traveling normally outward from the page (toward the reader) is the resultant of two elliptically polarized waves, one with components of E given by:

$$\mathscr{E}'_y = 3\cos\omega t$$

$$\mathscr{E}'_x = 7\cos\left(\omega t + \frac{\pi}{2}\right)$$

And the other with components given by:

$$\mathscr{E}_y'' = 2\cos\omega t$$

$$\mathscr{E}_x'' = 3\cos\left(\omega t - \frac{\pi}{2}\right)$$

(a) What is the axial ratio of the resultant wave?
(b) Does the resultant vector E rotate clockwise or counterclockwise?

(a) $E_y = E_y' + E_y'' = 3\cos\omega t + 2\cos\omega t = 5\cos\omega t$

$E_x = E_x' + E_x'' = 7\cos(\omega t + \frac{\pi}{2}) + 3\cos(\omega t - \frac{\pi}{2})$

$= -7\sin\omega t + 3\sin\omega t = -4\sin\omega t$

$AR = \frac{5}{4} = 1.25$

(b) At $\omega t = 0$, $\vec{E} = 5\hat{a}_y$

At $\omega t = \pi/2 \Rightarrow \vec{E} = -4\hat{a}_x \Rightarrow$ Rotation in CCW

5. Design an antenna with omnidirectional amplitude pattern with a half-power beam width of 90°, Express its radiation intensity by $U = \sin^n\theta$. Determine the value of n and attempt to identify elements that exhibit such a pattern. Determine the directivity of the antenna.

Solution: Since the half-power beamwidth is 90°, the angle at which the half-power point occurs is $\theta = 45°$. Thus

$$U(\theta = 45°) = 0.5 = \sin^n(45°) = (0.707)^n$$

or

$$n = 2$$

$$U_{max} = 1$$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} \sin^2\theta \, \sin\theta \, d\theta \, d\phi = \frac{8\pi}{3}$$

$$D_0 = \frac{4\pi}{8\pi/3} = \frac{3}{2} = 1.761 \text{ dB}$$

6. The normalized far-zone field pattern of an antenna is given by

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$$E = \begin{cases} (\sin\theta\cos^2\phi)^{1/2} & 0 \le \theta \le \pi \text{ and } 0 \le \phi \le \pi/2, 3\pi/2 \le \phi \le 2\pi \\ 0 & \text{elsewhere} \end{cases}$$

Find the directivity using
(a) The exact expression
(b) Kraus' approximate formula

$$U = \frac{1}{2\eta}|E|^2 = \frac{1}{2\eta}\sin\theta\cos^2\phi \Rightarrow U_{max} = \frac{1}{2\eta}$$

(a). $P_{rad} = 2\cdot\int_0^{\pi/2}\int_0^{\pi} \frac{1}{2\eta}\sin^2\theta\cos^2\phi \, d\theta \, d\phi = \frac{1}{\eta}\left(\frac{\pi}{4}\right)\left(\frac{\pi}{2}\right) = \frac{\pi^2}{8\eta}$

$$D_0 = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi\left(\frac{1}{2\eta}\right)}{\frac{\pi^2}{8\eta}} = \frac{16}{\pi} = 5.09 = 7.07\,dB$$

(b). $U_{max} = \frac{1}{2\eta}$ at $\theta = \pi/2$, $\phi = 0$

In the elevation plane through the maximum $\phi = 0$ and $U = \frac{1}{2\eta}\sin\theta$.
The 3-dB point occurs when
$U = 0.5\,U_{max} = 0.5\left(\frac{1}{2\eta}\right) = \frac{1}{2\eta}\sin\theta_1 \Rightarrow \theta_1 = \sin^{-1}(0.5) = 30^\circ$
Therefore $\Theta_{1d} = 2(90-30) = 120^\circ$

In the azimuth plane through the maximum $\theta = \pi/2$ and $U = \frac{1}{2\eta}\cos^2\phi$.
The 3-dB point occurs when $U = 0.5\,U_{max} = 0.5\left(\frac{1}{2\eta}\right) = \frac{1}{2\eta}\cos^2\theta_1 \Rightarrow$
$\phi_1 = \cos^{-1}(0.707) = 45^\circ$, $\Theta_{2d} = 2(90^\circ - 45^\circ) = 90^\circ$.

Therefore using Kraus' formula $D_0 \simeq \frac{41{,}253}{120\cdot(90)} = 3.82 = 5.82\,dB$

REPORT

1. The normalized radiation intensity of an antenna is represented by

$$U(\theta) = \cos^2(\theta)\cos^2(3\theta), \quad (0 \le \theta \le 90^\circ, \quad 0^\circ \le \phi \le 360^\circ)$$

Find the exact and approximate directivity.

2. The radiation intensity is represented by

$$U = \begin{cases} U_0 \sin(\pi\sin\theta), & 0 \le \theta \le \pi/2 \text{ and } 0 \le \phi \le 2\pi \\ 0 & \text{elsewhere} \end{cases}$$

Find θ_{HP} and draw the radiation pattern.

Good Luck

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